

# Systematic entanglement concentration for unknown less-entangled three-photon W states\*

Fang-Fang Du<sup>1</sup> and Fu-Guo Deng<sup>1,2,†</sup>

<sup>1</sup>*Department of Physics, Applied Optics Beijing Area Major Laboratory,  
Beijing Normal University, Beijing 100875, China*

<sup>2</sup>*State key Laboratory of Networking and Switching Technology,  
Beijing University of Posts and Telecommunications, Beijing 100876, China*

(Dated: November 3, 2015)

We present a systematic entanglement concentration protocol (ECP) for an arbitrary unknown less-entangled three-photon W state, resorting to the optical property of the quantum-dot spins inside one-sided optical microcavities. In our ECP, the parties obtain not only some three-photon systems in the partially entangled with two unknown parameters when one of the parties picks up the robust odd-parity instance with the parity-check gate (PCG) on his two photons, but also some entangled two-photon systems by keeping the even-parity instance in the first step. By exploiting the above three-photon and two-photon systems with the same parameters as the resource for the second step of our ECP, the parties can obtain a standard three-photon W state by keeping the robust odd-parity instance. Meanwhile, the systems in the even-parity instance can be used as the resource in the next round of our ECP. The success probability of our ECP is largely increased by iteration of the ECP process. As it does require that all the coefficients are unknown for the parties, our ECP maybe have good applications in quantum communication network in future.

PACS numbers: 03.67.Pp, 03.67.Bg, 03.67.Hk, 42.50.Pq

## I. INTRODUCTION

Quantum communication becomes one of the two important branches of quantum information and it has attracted much attention, especially teleportation [1], dense coding [2, 3], quantum key distribution (QKD) [4–7], quantum secret sharing [8], and quantum secure direct communication [9–11]. Quantum entanglement is a key resource for quantum communication. It can act as the information carrier in some important quantum communication protocols [5–10]. In practical long-distance quantum communication, quantum repeaters are required to overcome the photon loss and decoherence from environment noise, in which entanglement is used to create the quantum channel between two neighboring quantum nodes. That is to say, all the tasks in long-distance quantum communication should require high-fidelity entanglement. However, entanglement can only be produced locally and it is very fragile in the process of transmission and storage, due to the influence of decoherence and the imperfection at the source. The decoherence of entangled quantum systems will decrease the security of QKD protocols and even makes a quantum teleportation and a quantum dense coding protocol fail.

There are some useful methods to depress the above fragile effect on entangled photon systems, such as conventional entanglement purification [12–15] with which the remote users can obtain some high-fidelity entangled systems from low-fidelity ones, deterministic entangle-

ment purification [16–21] (which works in a completely deterministic way, not in a probabilistic way, and they can reduce the quantum resource sacrificed largely [22]), and entanglement concentration [23–36]. By definition, entanglement purification protocols are used to distill some high-fidelity entangled photon systems from a less-entangled ensemble in a mixed entangled state [12–21] while entanglement concentration protocols (ECPs) [23–42] are used to obtain a subset of photon systems in a maximally entangled state from a set of systems in a partially entangled pure state. The former is more general as a photon system is usually in a mixed entangled state after it is transmitted over a noisy channel, but the latter is a more efficient in some particular cases, such as those with decoherence of entanglement arising from the storage process or the imperfect entanglement source.

The first ECP was proposed by Bennett *et al* [23] in 1996. In 2001, two ECPs based on polarizing beam splitters were proposed [24, 25]. In 2008, Sheng, Deng, and Zhou [26] proposed a repeatable ECP which has a far higher efficiency and yield than the PBS-ECPs [24, 25], by iteration of the entanglement concentration process three times. In fact, depending on whether the parameters of the less-entangled states are known [27–31] or not [23–26], the ECPs can be classed into two groups. When the parameters are known, one nonlocal photon system is enough for entanglement concentration [27–31], far more efficient than those with unknown parameters [23–26]. In 2013, Ren, Du, and Deng [31] proposed the parameter-splitting method to extract the maximally entangled photons in both the polarization and spatial degrees of freedom (DOFs) when the coefficients of the initial partially hyperentangled states are known. This fascinating method is very efficient and simple in terms

\*Published in Laser Phys. Lett. **12**, 115202 (2015).

†Corresponding author: fgdeng@bnu.edu.cn

of concentrating partially entangled states, and it can be achieved with the maximum success probability by performing the protocol only once, resorting to linear optical elements only, not nonlinearity [43]. They [31] also gave the first hyperentanglement concentration protocol (hyper-ECP) for the known and unknown polarization-spatial less-hyperentangled states with linear-optical elements only. Recently, some good hyper-ECPs [32–36] for photon systems were proposed. Now, some efficient ECPs for atomic systems [37] and electronic systems [38–41] have been proposed.

By far, there are few ECPs for entangled pure W-class states [44–52]. In essence, W states are inequivalent to Greenberger-Horne-Zeilinger (GHZ) states, as they cannot be converted into each other under stochastic local operations and classical communication (LOCC). Moreover, tripartite W entanglement has both bipartite and tripartite quantum entanglement simultaneously, thus it is robust to the loss of one qubit [53] and can be used in some quantum information processing such as unconditionally teleclone coherent states [54], probabilistic teleportation of unknown atomic state [55], quantum information sharing [56], random-party distillation [57, 58], and so on. That is, it is of practical significance to discuss the entanglement concentration on the partially entangled W state. In 2010, Yildiz [44] proposed an optimal distillation of three-qubit asymmetric W states. Based on linear optical elements, an ECP for partially entangled W states are proposed by Wang *et al* [45]. Subsequently, Du *et al* [46] and Gu *et al* [47] improved the ECP for the special W states by exploiting the cross-Kerr nonlinearity, respectively. However, these ECPs [45–47] are used to deal with the concentration on three-photon systems in a partially entangled W state with only two parameters, not arbitrary coefficients. In 2012, Sheng *et al* [48] proposed an ECP for three-photon systems in an arbitrary less-entangled W-type state with the known coefficients by assisting two different specially polarized single-photon sources. Based on linear optical elements, Wang and Long [49] presented two three-photon ECPs for an arbitrary unknown less-entangled W-class state in 2013.

Recently, a single spin coupled to an optical microcavity based on a charged self-assembled GaAs/InAs quantum dot (QD) has attracted much attention as it is a novel candidate for a quantum qubit. Since Hu *et al* [59] pointed out that the interaction between a circularly polarized light and a QD-cavity system can be used for quantum information processing, some ECPs [33, 38, 39, 51] have been proposed with this system. For example, in 2011, Wang *et al* [39] proposed an ECP based on QD-microcavity systems with two copies of partially entangled two-electron systems to obtain a maximally entangled two-electron system probabilistically. Subsequently, Wang [38] showed that each two-electron-spin system in a partially entangled state can be concentrated with the assistance of an ancillary quantum dot and a single photon, not two copies of two-electron spin systems.

In 2013, Sheng *et al* [51] proposed an efficient ECP for W-class states assisted by the double-sided optical microcavities.

In this paper, we propose a systematic ECP for an arbitrary unknown less-entangled three-photon W state, resorting to the optical property of the quantum-dot spins inside one-sided optical microcavities. The parties obtain not only some partially entangled three-photon systems with two unknown parameters when one of the parties picks up the robust odd-parity instance with the parity-check gate (PCG) on his two photons, but also some entangled two-photon systems by keeping an even-parity instance in the first step of the first round of concentration. By exploiting the above three-photon and two-photon systems with the same parameters as the resource for the second step of our ECP, the parties can obtain a standard three-photon W state by keeping the robust odd-parity instance, far different to the previous ECPs for W-class states with unknown parameters [44]. Meanwhile, the systems in the even-parity instance can be used as the resource in the next round of the ECP. The success probability of our ECP is largely increased by iteration of the ECP process. Besides, as the side leakage and cavity loss may be difficult to control or reduce for the photonic qubits in the double-sided QD-cavity system, our ECP is relatively easier to be implemented in experiment than the ECP with a double-sided QD-cavity system [51]. These advantages maybe make our ECPs more useful in quantum communication network in future.

## II. PARITY-CHECK GATE ON TWO PHOTONS ASSISTED BY A QD-CAVITY SYSTEM

### A. Interaction between a circularly polarized light and a QD-cavity system

The solid-state system discussed here is a singly charged QD in a one-sided cavity, e.g., a self-assembled In(Ga)As QD or a GaAs interface QD located in the center of an optical resonant cavity (the bottom distributed Bragg reflectors are 100% reflective and the top distributed Bragg reflectors are partially reflective) to achieve a maximal light-matter coupling [59], shown in Fig. 1(b). If an excess electron is injected into the QD, the optical resonance can create the negatively charged exciton  $X^-$  that consists of two electrons bound to one hole [60]. This means that  $X^-$  has the spin-dependent optical transitions [61] (shown in Fig. 1(a)) for the circularly polarized probe lights, according to Pauli's exclusion principle. The left-circularly polarized light  $|L\rangle$  is resonantly absorbed to create the negatively charged exciton  $|\uparrow\downarrow\uparrow\rangle$  for the excess electron spin state  $|\uparrow\rangle_e$ , and the right-circularly polarized light  $|R\rangle$  is resonantly absorbed to create the negatively charged exciton  $|\downarrow\downarrow\downarrow\rangle$  for the excess electron spin state  $|\downarrow\rangle_e$ . Here  $|\uparrow\rangle$  ( $|\downarrow\rangle$ ) represents the heavy-hole spin state  $|+\frac{3}{2}\rangle$  ( $|-\frac{3}{2}\rangle$ ). This process can be described by the Heisenberg equations for

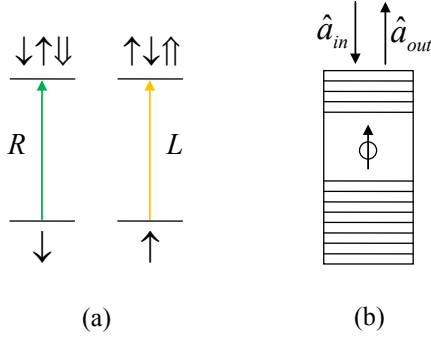


FIG. 1: The spin-dependent transitions for negatively charged exciton  $X^-$ . (a) Spin selection rule for optical transitions of negatively charged exciton  $X^-$  due to the Pauli's exclusion principle. (b) A charged QD inside a micropillar microcavity with circular cross section. L and R represent the left and the right circularly polarized lights, respectively.  $\uparrow$  and  $\downarrow$  represent the spins of the excess electron.  $\downarrow\uparrow\downarrow$  and  $\uparrow\downarrow\uparrow$  represent the negatively charged exciton  $X^-$ .

the cavity field operator  $a$  and  $X^-$  dipole operator  $\sigma_-$  in the interaction picture ( $\hbar = 1$ ) [62],

$$\begin{aligned} \frac{da}{dt} &= -\left[i(\omega_c - \omega) + \frac{\kappa}{2} + \frac{\kappa_s}{2}\right]a - g\sigma_- - \sqrt{\kappa}a_{in}, \\ \frac{d\sigma_-}{dt} &= -\left[i(\omega_{X^-} - \omega) + \frac{\gamma}{2}\right]\sigma_- - g\sigma_z a, \\ a_{out} &= a_{in} + \sqrt{\kappa}a. \end{aligned} \quad (1)$$

Here,  $\omega$ ,  $\omega_{X^-}$ , and  $\omega_c$  are the frequencies of the input probe light,  $X^-$  transition, and cavity mode, respectively.  $\frac{\gamma}{2}$  and  $\frac{\kappa}{2}$  are the decay rates of  $X^-$  and the cavity field, respectively.  $\frac{\kappa_s}{2}$  is the side leakage rate of the cavity.  $g$  is the coupling strength between  $X^-$  and the cavity mode.

Considering a weak excitation condition with  $X^-$  staying in the ground state at most time and  $\langle\sigma_z\rangle = 1$ , the reflection coefficient of circularly polarized light after interacting with a QD-cavity system is [59, 62]

$$r_h(\omega) = 1 - \frac{\kappa \left[ i(\omega_{X^-} - \omega) + \frac{\gamma}{2} \right]}{\left[ i(\omega_{X^-} - \omega) + \frac{\gamma}{2} \right] \left[ i(\omega_c - \omega) + \frac{\kappa}{2} + \frac{\kappa_s}{2} \right] + g^2}. \quad (2)$$

If the  $X^-$  is uncoupled to the cavity, which is called a cold cavity with the coupling strength  $g = 0$ , the reflection coefficient becomes [59, 62]

$$r_0(\omega) = \frac{i(\omega_c - \omega) - \frac{\kappa}{2} + \frac{\kappa_s}{2}}{i(\omega_c - \omega) + \frac{\kappa}{2} + \frac{\kappa_s}{2}}. \quad (3)$$

The polarized light may have a phase shift after being reflected from the QD-cavity system. By adjusting the frequencies  $\omega$  and  $\omega_c$  and neglecting the cavity side leakage, one can get  $|r_0(\omega)| \cong 1$  for a cold cavity and  $|r_h(\omega)| \cong 1$  for a hot cavity. The  $|L\rangle$  light gets a phase shift of  $\varphi_h$  for

a hot cavity when the excess electron spin state is  $|\uparrow\rangle_e$ , and it gets a phase shift of  $\varphi_0$  for a cold cavity when the excess electron spin state is  $|\downarrow\rangle_e$ . Conversely, the  $|R\rangle$  light gets a phase shift of  $\varphi_0$  for a cold cavity when the excess electron spin state is  $|\uparrow\rangle_e$ , and it gets a phase shift of  $\varphi_h$  for a hot cavity when the excess electron spin state is  $|\downarrow\rangle_e$ . Therefore, the superposition of two circularly polarized probe beams  $(|R\rangle + |L\rangle)/\sqrt{2}$  becomes  $(e^{i\varphi_0}|R\rangle + e^{i\varphi_h}|L\rangle)/\sqrt{2}$  for the electron spin state  $|\uparrow\rangle_e$  and  $(e^{i\varphi_h}|R\rangle + e^{i\varphi_0}|L\rangle)/\sqrt{2}$  for the electron spin state  $|\downarrow\rangle_e$  after being reflected from the QD-cavity system. The Faraday rotation is defined by the rotation angle of the polarization direction  $\theta_F^\uparrow = (\varphi_0 - \varphi_h)/2 = \theta_F^\downarrow$ . If a polarized probe beam  $(|R\rangle + |L\rangle)/\sqrt{2}$  is put into the QD-cavity system with the electron spin in the state  $(\alpha|\uparrow\rangle + \beta|\downarrow\rangle)_e$ , after reflection, the state of the system composed of the light and the electron spin becomes entangled,

$$\begin{aligned} &(|R\rangle + |L\rangle)/\sqrt{2} \otimes (\alpha|\uparrow\rangle + \beta|\downarrow\rangle)_e \longrightarrow \\ &e^{i\varphi_0}[\alpha(|R\rangle + e^{i\Delta\varphi}|L\rangle)|\uparrow\rangle_e + \beta(e^{i\Delta\varphi}|R\rangle + |L\rangle)|\downarrow\rangle_e] / \sqrt{2}. \end{aligned} \quad (4)$$

Here,  $\Delta\varphi = \varphi_h - \varphi_0$ ,  $\varphi_0 = \arg[r_0(\omega)]$  and  $\varphi = \arg[r(\omega)]$ . Due to the spin-selection rule,  $|R\rangle$  and  $|L\rangle$  lights encounter different phase shifts after being reflected from the one-sided QD-cavity system, and the state of the system composed of the light and the spin in QD becomes entangled.

If the frequencies of the input light and cavity mode are adjusted as  $\omega - \omega_c \approx \kappa/2$ , the relative phase shift of the left and right circularly polarized lights is  $\Delta\phi = \frac{\pi}{2}$ . The rules of the input photon states changing under the interaction of the photon and the QD-cavity system can be described as follows:

$$\begin{aligned} |L, \uparrow\rangle &\rightarrow |L, \uparrow\rangle, & |L, \downarrow\rangle &\rightarrow i|L, \downarrow\rangle, \\ |R, \uparrow\rangle &\rightarrow i|R, \uparrow\rangle, & |R, \downarrow\rangle &\rightarrow |R, \downarrow\rangle. \end{aligned} \quad (5)$$

## B. Parity-check gate on the polarizations of two photons

The principle of our parity-check gate on the polarizations of two photons is shown in Fig.2. Suppose that the electron spin in the QD is prepared in the superposition state  $|\psi\rangle_e^s = \frac{1}{\sqrt{2}}(|\uparrow\rangle + |\downarrow\rangle)_e$ . Photon 1 in the state  $|\psi\rangle_1^p = \alpha_1|R\rangle_1 + \beta_1|L\rangle_1$  and photon 2 in the state  $|\psi\rangle_2^p = \frac{1}{\sqrt{2}}(|R\rangle_2 + |L\rangle_2)$  enter into the QD-cavity system in sequence. An optical circulator (OC) first directs photon 1 until it is reflected by the cavity, and then it is switched for photon 2. The time difference between photons 1 and 2 should be less than the spin coherence time. After two photons are reflected, the state of the composite system composed of two photons and one QD-spin is

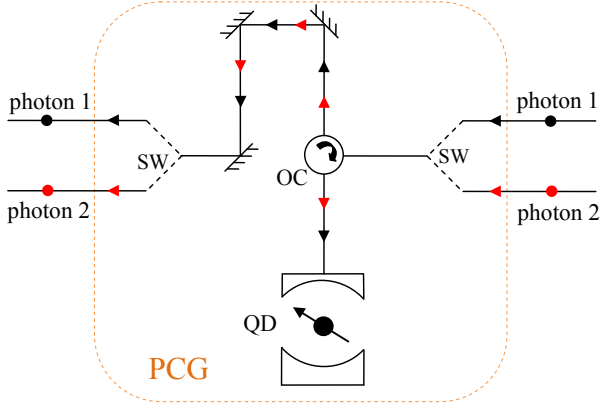


FIG. 2: Schematic diagram of the parity-check gate on the polarizations of two photons, assisted by a QD-spin in a single-sided microcavity with the top mirror partially reflective and the bottom mirror 100% reflective. The optical switch (SW) and the optical circulator (OC) make photon 1 and photon 2 pass through the cavity in sequence.

evolved as follows:

$$\begin{aligned}
 |S\rangle_0 &\equiv |\psi\rangle_1^p \otimes |\psi\rangle_2^p \otimes |\psi\rangle_e^s \rightarrow \\
 |S\rangle_1 &= \frac{1}{2}(|\uparrow\rangle - |\downarrow\rangle)_e (\alpha_1|R\rangle_1|R\rangle_2 - \beta_1|L\rangle_1|L\rangle_2) \\
 &\quad + \frac{i}{2}(|\uparrow\rangle + |\downarrow\rangle)_e (\alpha_1|R\rangle_1|L\rangle_2 + \beta_1|L\rangle_1|R\rangle_2).
 \end{aligned} \quad (6)$$

By detecting the electron-spin state, one can distinguish the even-parity state  $|\phi_{12}\rangle_{\text{even}} = \alpha_1|R\rangle_1|R\rangle_2 - \beta_1|L\rangle_1|L\rangle_2$  of the two-photon system from the odd-parity state  $|\psi_{12}\rangle_{\text{odd}} = \alpha_1|R\rangle_1|L\rangle_2 + \beta_1|L\rangle_1|R\rangle_2$ . This task can be achieved with a probe photon 3 which interacts with the electron spin (the GFR-based quantum nondemolition method) [63]. In detail, suppose that the photon 3 is initially in the state  $|\psi\rangle_3^p = \frac{1}{\sqrt{2}}(|R\rangle_3 + |L\rangle_3)$ . One performs a Hadamard transformation [ $|\uparrow\rangle_e \rightarrow |+\rangle_e = \frac{1}{\sqrt{2}}(|\uparrow\rangle + |\downarrow\rangle)_e$ ,  $|\downarrow\rangle_e \rightarrow |-\rangle_e = \frac{1}{\sqrt{2}}(|\uparrow\rangle - |\downarrow\rangle)_e$ , e.g., using a  $\pi/2$  microwave pulse] on the electron spin before photon 3 is input into the cavity (the photon 3 has the same frequency as the photons 1 and 2). After the photon 3 is reflected, the state of the composite system composed of the three photons and one electron spin becomes

$$\begin{aligned}
 |S\rangle_2 &= \frac{1}{2}(|L\rangle_3 + i|R\rangle_3)|\downarrow\rangle_e (\alpha_1|R\rangle_1|R\rangle_2 - \beta_1|L\rangle_1|L\rangle_2) \\
 &\quad - \frac{1}{2}(|L\rangle_3 - i|R\rangle_3)|\uparrow\rangle_e (\alpha_1|R\rangle_1|L\rangle_2 + \beta_1|L\rangle_1|R\rangle_2).
 \end{aligned} \quad (7)$$

The output state of photon 3 can be measured in orthogonal linear polarizations. If photon 3 is detected in the  $(|L\rangle_3 + i|R\rangle_3)/\sqrt{2}$  state ( $45^\circ$  linear), the electron spin is in the state  $|\downarrow\rangle_e$  and the system composed of photon 1 and photon 2 is in the even-parity state  $|\phi_{12}\rangle_{\text{even}} = \alpha_1|R\rangle_1|R\rangle_2 - \beta_1|L\rangle_1|L\rangle_2$ . Otherwise,

if photon 3 is detected in the  $(|L\rangle_3 - i|R\rangle_3)/\sqrt{2}$  state ( $-45^\circ$  linear), the electron spin is in the state  $|\uparrow\rangle_e$  and the two-photon system is in the odd-parity state  $|\psi_{12}\rangle_{\text{odd}} = \alpha_1|R\rangle_1|L\rangle_2 - \beta_1|L\rangle_1|R\rangle_2$ .

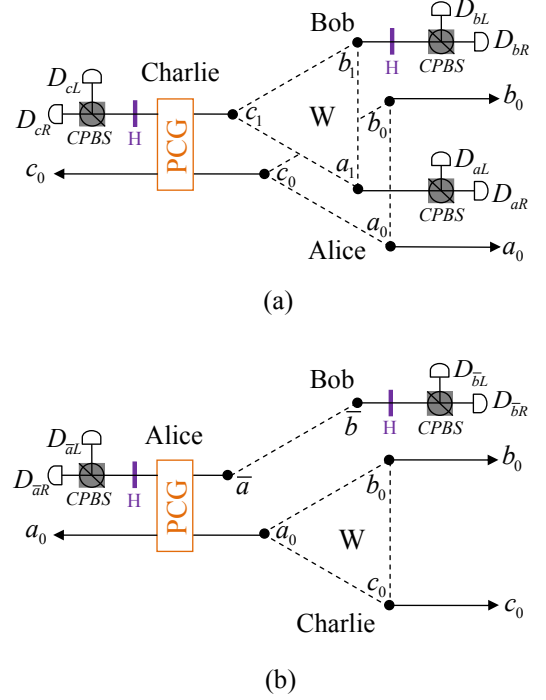


FIG. 3: Schematic diagram of our ECP for three-photon systems in a less-entangled W state with the parity-check gate (PCG) on the polarizations of two photons. (a) The first step of our ECP. (b) The second step of our ECP. CPBS represents a polarizing beam splitter in the circularly polarized basis  $\{|R\rangle, |L\rangle\}$ , which transmits the  $|R\rangle$  polarization photons and reflects the  $|L\rangle$  polarization photons. H represents a Hadamard operation on the polarization of the photon.  $D_{mL}$  and  $D_{mR}$ , ( $m \in \{a, b, c, \bar{a}, \bar{b}\}$ ) are single-photon detectors.

### III. SYSTEMATIC ECP FOR LESS-ENTANGLED THREE-PHOTON W STATE

Let us assume that the three-photon system  $a_i b_i c_i$  ( $i = 0, 1, 2, \dots$ ) is in the following less-entangled state:

$$\begin{aligned}
 |\Phi\rangle_{a_i b_i c_i} &= \alpha |R\rangle_{a_i} |R\rangle_{b_i} |L\rangle_{c_i} + \beta |R\rangle_{a_i} |L\rangle_{b_i} |R\rangle_{c_i} \\
 &\quad + \gamma |L\rangle_{a_i} |R\rangle_{b_i} |R\rangle_{c_i},
 \end{aligned} \quad (8)$$

where the subscripts  $a_i$ ,  $b_i$ , and  $c_i$  represent the three photons belonging to the three remote parties in quantum communication, say Alice, Bob, and Charlie, respectively. Here  $\alpha$ ,  $\beta$ , and  $\gamma$  are three arbitrary unknown parameters and they satisfy the relation  $|\alpha|^2 + |\beta|^2 + |\gamma|^2 = 1$ . The principle of our systematic ECP with nonlinear optical elements is shown in Fig.3, and it includes two steps. We describe it in detail as follows.

The principle for the first step of our systematic ECP is shown in Fig.3 (a). In this step, in each round of entanglement concentration, Alice, Bob and Charlie operate two pairs of three-photon less-entangled systems, say  $a_0b_0c_0$  and  $a_1b_1c_1$ . The state  $|\Phi\rangle_{a_0b_0c_0} \otimes |\Phi\rangle_{a_1b_1c_1}$  ( $i = 0, 1$ ) of the composite system composed of the six photons can be rewritten as

$$\begin{aligned} |\Psi_1\rangle = & \alpha|RR\rangle_{a_0b_0}(\beta|RL\rangle + \gamma|LR\rangle)_{a_1b_1}|LR\rangle_{c_0c_1} \\ & + \alpha(\beta|RL\rangle + \gamma|LR\rangle)_{a_0b_0}|RR\rangle_{a_1b_1}|RL\rangle_{c_0c_1} \\ & + \alpha^2|RR\rangle_{a_0b_0}|RR\rangle_{a_1b_1}|LL\rangle_{c_0c_1} + (\beta|RL\rangle \\ & + \gamma|LR\rangle)_{a_0b_0}(\beta|RL\rangle + \gamma|LR\rangle)_{a_1b_1}|RR\rangle_{c_0c_1}. \end{aligned} \quad (9)$$

Charlie lets her two photons  $c_0$  and  $c_1$  pass through PCG. The outcomes will be divided into two groups, the odd-parity one and the even-parity one.

If the outcome of the PCG by Charlie is an odd-parity one, the state of the composite system changes from  $|\Psi_1\rangle$  to the state (without normalization)

$$\begin{aligned} |\Psi_{1o}\rangle = & \alpha(\beta|RL\rangle + \gamma|LR\rangle)_{a_0b_0}|RR\rangle_{a_1b_1}|RL\rangle_{c_0c_1} \\ & + \alpha|RR\rangle_{a_0b_0}(\beta|RL\rangle + \gamma|LR\rangle)_{a_1b_1}|LR\rangle_{c_0c_1}. \end{aligned} \quad (10)$$

Subsequently, Charlie informs Alice to measure her photon  $a_1$  with the basis  $\{|R\rangle, |L\rangle\}$ . If Alice gets the outcome  $|R\rangle_{a_1}$  (the single-photon detector  $D_{aR}$  is clicked), the state  $|\Psi_{1o}\rangle$  of the composite system becomes

$$\begin{aligned} |\Psi'_{1o}\rangle = & \nu_1[\beta|RRL\rangle_{a_0b_0c_0}|LR\rangle_{b_1c_1} + (\beta|RLR\rangle \\ & + \gamma|LRR\rangle)_{a_0b_0c_0}|RL\rangle_{b_1c_1}] \end{aligned} \quad (11)$$

with the probability  $P_{1o} = |\alpha|^2(|\gamma|^2 + 2|\beta|^2)$ . Here  $\nu_1 = \frac{1}{\sqrt{|\gamma|^2 + 2|\beta|^2}}$ . After performing the Hadamard operations  $H$  on the photons  $b_1$  and  $c_1$ , Bob and Charlie measure their photons  $b_1$  and  $c_1$  with the basis  $\{|R\rangle, |L\rangle\}$ . When Charlie and Bob obtain an even-parity one (i.e.,  $|RR\rangle_{b_1c_1}$  or  $|LL\rangle_{b_1c_1}$ ), the three-photon system  $a_0b_0c_0$  is in the state

$$|\Phi_{1o}^+\rangle = \nu_1(\beta|RRL\rangle + \beta|RLR\rangle + \gamma|LRR\rangle)_{a_0b_0c_0}. \quad (12)$$

When Charlie and Bob obtain an odd-parity one (i.e.,  $|RL\rangle_{b_1c_1}$  or  $|LR\rangle_{b_1c_1}$ ), the system is in the state

$$|\Phi_{1o}^-\rangle = \nu_1(-\beta|RRL\rangle + \beta|RLR\rangle + \gamma|LRR\rangle)_{a_0b_0c_0}. \quad (13)$$

With a phase-flip operation  $\sigma_z = |R\rangle\langle R| - |L\rangle\langle L|$  on the photon  $c_0$ , Charlie can transform the state  $|\Phi_{1o}^-\rangle$  into the state  $|\Phi_{1o}^+\rangle$ .

If the outcome of the PCG by Charlie is an even-parity one, the composite system collapses from the state  $|\Psi_1\rangle$  to (without normalization)

$$\begin{aligned} |\Psi_{1e}\rangle = & -\alpha^2|RR\rangle_{a_0b_0}|RR\rangle_{a_1b_1}|LL\rangle_{c_0c_1} + (\beta|RL\rangle \\ & + \gamma|LR\rangle)_{a_0b_0}(\beta|RL\rangle + \gamma|LR\rangle)_{a_1b_1}|RR\rangle_{c_0c_1}. \end{aligned} \quad (14)$$

Subsequently, Charlie measures his photons  $c_0$  and  $c_1$  with the basis  $\{|R\rangle, |L\rangle\}$ . When Charlie gets the measurement outcome  $|RR\rangle_{c_0c_1}$ , the state  $|\Psi_{1e}\rangle$  collapses into

$$|\Psi'_{1e}\rangle = (\beta|RL\rangle + \gamma|LR\rangle)_{a_0b_0}(\beta|RL\rangle + \gamma|LR\rangle)_{a_1b_1}. \quad (15)$$

In this time, Alice and Bob can obtain two pairs of two-photon systems in the state  $|\Phi_{1e}\rangle = \frac{1}{\sqrt{|\gamma|^2 + |\beta|^2}}(\beta|RL\rangle + \gamma|LR\rangle)_{\bar{a}\bar{b}}$  with the probability  $P_{1e} = (|\gamma|^2 + |\beta|^2)^2$ .

The principle for the second step of our systematic ECP is shown in Fig.3 (b). In this step, Alice, Bob and Charlie exploit a set of the three-photon systems in the state  $|\Phi_{1o}^+\rangle$  and the two-photon systems in the state  $|\Phi_{1e}\rangle$  to finish the task of entanglement concentration for obtaining a subset of the three-photon systems in a standard W state. To this end, Alice lets her two photons  $a_0$  and  $\bar{a}$  go through the PCG. If the photon pair  $a_0\bar{a}$  is in the odd-parity state, the five-photon system is in the state  $|\Psi_{2o}\rangle$  in which each item has the same parameter, that is,

$$\begin{aligned} |\Psi_{2o}\rangle = & \frac{1}{\sqrt{3}}[|RL\rangle_{a_0\bar{a}}|R\rangle_{\bar{b}}(|RL\rangle + |LR\rangle)_{b_0c_0} \\ & + |LR\rangle_{a_1\bar{a}}|L\rangle_{\bar{b}}|RR\rangle_{b_0c_0}] \end{aligned} \quad (16)$$

with the probability  $P'_{1o} = \frac{3|\gamma|^2|\beta|^2}{(|\gamma|^2 + 2|\beta|^2)(|\gamma|^2 + |\beta|^2)}$ . Then Alice and Bob perform Hadamard operation and measure the two photons  $\bar{a}$  and  $\bar{b}$ . If Alice and Bob obtain an even-parity outcome, the three-photon system  $a_0b_0c_0$  is in the standard three-photon W state

$$|W^+\rangle = \frac{1}{\sqrt{3}}(|RRL\rangle + |RLR\rangle + |LRR\rangle)_{a_0b_0c_0}. \quad (17)$$

If they obtain an odd-parity one, the three-photon system is in the state  $|W^-\rangle$

$$|W^-\rangle = \frac{1}{\sqrt{3}}(|RRL\rangle + |RLR\rangle - |LRR\rangle)_{a_0b_0c_0}. \quad (18)$$

With a phase-flip operation  $\sigma_z$  on the photon  $a_0$ , Alice can transform the state  $|W^-\rangle$  into the state  $|W^+\rangle$ . Therefore, Alice, Bob, and Charlie can get a three-photon system in a standard  $|W^+\rangle$  state from the three-photon systems in a partially entangled state  $|\Phi\rangle_{a_i b_i c_i}$  with the success probability  $P_1 = \xi P'_{1o}$ , where  $\xi = \min\{P_{1o}, P_{1e}\}$ . If there are enough three-photon unknown W states and they satisfy  $P_{1o} = P_{1e}$ , three participants can obtain the standard W state with the success probability is  $P_1 = \frac{3|\alpha|^2|\beta|^2|\gamma|^2}{|\gamma|^2 + |\beta|^2}$  in the first round of concentration.

If the photon pair  $a_0\bar{a}$  is in the even-parity one, the five-photon system is in the state

$$\begin{aligned} |\Psi_{2e}\rangle = & \nu_2[\beta^2|RR\rangle_{a_0\bar{a}}|L\rangle_{\bar{b}}(|RL\rangle + |LR\rangle)_{b_0c_0} \\ & - \gamma^2|LL\rangle_{a_0\bar{a}}|R\rangle_{\bar{b}}|RR\rangle_{b_0c_0}] \end{aligned} \quad (19)$$

with the probability  $P'_{1e} = \frac{|\gamma|^4 + 2|\beta|^4}{(|\gamma|^2 + 2|\beta|^2)(|\gamma|^2 + |\beta|^2)}$ . Here  $\nu_2 = \frac{1}{\sqrt{|\gamma|^4 + 2|\beta|^4}}$ . Similar to the above discussion of the measurement results of the odd-parity case (the state  $|\Psi_{2e}\rangle$ ), three participants obtain the states  $|\Phi_{2e}^+\rangle$  and  $|\Phi_{2e}^-\rangle$  when the outcomes obtained by Alice and Bob are an odd-parity one and an even-parity one, respectively. Alice can transform the state  $|\Phi_{2e}^-\rangle$  into the state  $|\Phi_{2e}^+\rangle$  by

performing a phase-flip operation  $\sigma_z$  on the photon  $a_0$ . Here

$$\begin{aligned} |\Phi_{2e}^+\rangle &= \nu_2(\beta^2|RRL\rangle + \beta^2|RLR\rangle + \gamma^2|LRR\rangle)_{a_0b_0c_0}, \\ |\Phi_{2e}^-\rangle &= \nu_2(\beta^2|RRL\rangle + \beta^2|RLR\rangle - \gamma^2|LRR\rangle)_{a_0b_0c_0}. \end{aligned} \quad (20)$$

It is not difficult to find that the state  $|\Phi_{2e}^+\rangle$  has the same form as the state  $|\Phi_{1o}^+\rangle$  shown in Eq.(12) but different parameters. We need only replace the parameters  $\beta$  and  $\gamma$  in Eq.(12) with the parameters  $\beta^2$  and  $\gamma^2$ , respectively. Obviously,  $|\Phi_{2e}^+\rangle$  is the resource for the ECP in the second round. The above discussion is the first round of our ECP. The detail for the procedure of the first round of our ECP for three-photon systems in an arbitrary W-type state with PCG is shown in Fig.4.

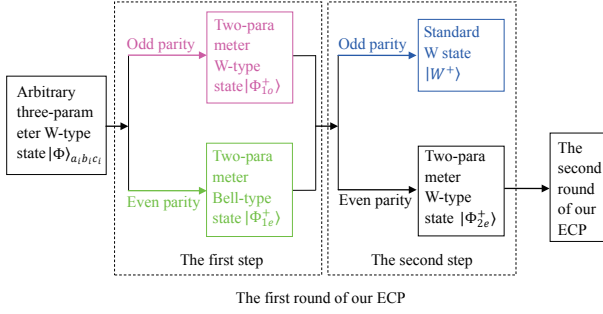


FIG. 4: Schematic diagram of the procedure of the first round of our ECP for three-photon systems in an arbitrary W-type state with PCG.

Now, we continue to discuss the second round of our ECP with the composite system composed of the six photons  $a_2b_2c_2$  and  $a_3b_3c_3$  (different from the previous discussion with six photons  $a_0b_0c_0$  and  $a_1b_1c_1$  in the first round) from a set of the three-photon systems in the state  $|\Phi_{2e}^+\rangle$ . First of all, we make the six photons  $a_2b_2c_2$  and  $a_3b_3c_3$  pass through the device shown in Fig. 3(a), only substituting  $a_2, a_3, b_2, b_3, c_2, c_3$  for  $c_0, c_1, b_0, b_1, a_0, a_1$ , respectively. Besides, it's worth noting that two three-photon systems do not go through the second step of the first round of our ECP.

Similar to the above discussion in the first round of our ECP, three parties obtain the standard three-photon W state with the success probability of  $P_{2o} = \frac{3|\beta|^4|\gamma|^4}{(|\gamma|^4+2|\beta|^4)^2}$ . By iterating the ECP several times, the success probability to get a maximally entangled W state from the initial partially entangled state is  $P_{no} = \frac{3|\beta|^{2n}|\gamma|^{2n}}{(2|\beta|^{2n}+|\gamma|^{2n})^2}$ , ( $n = 2, 3, \dots$ ) in the  $n$ -th round of our ECP, while the probability to obtain the partially entangled three-photon W states is  $P_{ne} = \frac{2|\beta|^{2n+1}+|\gamma|^{2n+1}}{(2|\beta|^{2n}+|\gamma|^{2n})^2}$ , ( $n = 2, 3, \dots$ ). Therefore, the total success probability to get a maximally entangled W state from the initial partially entangled state is

$$P = \xi P'_{1o} + \xi P'_{1e} P_{2o} + \dots + \xi P'_{1e} P_{2e} P_{3e} \dots P_{no}. \quad (21)$$

The total success probability of the maximally entangled W state vs  $\beta^2$  with  $P_{1o} = P_{1e}$  is shown in Fig. 5. It is quite clear that the total success probability gradually increases by iterating the ECP.

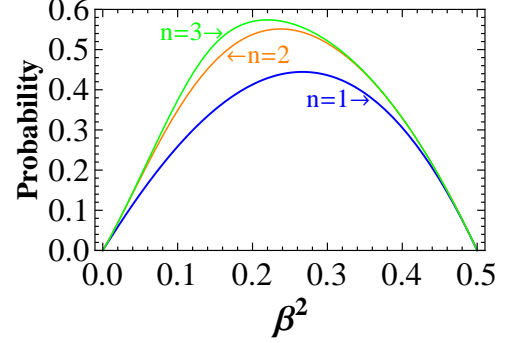


FIG. 5: The total success probability of the maximally entangled W state vs the parameter  $\beta^2$  with  $P_{1o} = P_{1e}$ , by iterating the ECP one time ( $n=1$ ), two times ( $n=2$ ), and three times ( $n=3$ ), respectively.

#### IV. DISCUSSION AND SUMMARY

So far, all the procedures in our scheme for the PCG are described in the case that the side leakage rate  $\kappa_s$  is negligible. To present our idea more realistically,  $\kappa_s$  should be taken into account. In this time, the rules of the input states changing under the interaction of the photon and the cavity become

$$\begin{aligned} |L, \uparrow\rangle &\rightarrow |r(\omega)||L, \uparrow\rangle, & |L, \downarrow\rangle &\rightarrow i|r_0(\omega)||L, \downarrow\rangle, \\ |R, \uparrow\rangle &\rightarrow i|r_0(\omega)||R, \uparrow\rangle, & |R, \downarrow\rangle &\rightarrow |r(\omega)||R, \downarrow\rangle. \end{aligned} \quad (22)$$

The fidelity and the efficiency of our PCG are sensitive to  $\kappa_s$  as  $\kappa_s$  influences the amplitudes of the reflected photon (see Eq.(5)). Here the fidelity of our PCG are defined as  $F = |\langle\psi_{real}|\psi_{ideal}\rangle|^2$ . Here,  $|\psi_{ideal}\rangle$  and  $|\psi_{real}\rangle$  are the final states in the ideal condition and in the realistic condition, respectively. The coupling strength  $g/(\kappa_s + \kappa) \cong 1.5$  [64] was reported in  $d = 1.5\mu m$  micropillar microcavities ( $Q \sim 8800$ ), and the coupling strength can be enhanced to  $g/(\kappa_s + \kappa) \cong 2.4$  ( $Q \sim 40000$ ) [65] by improving the sample designs, growth, and fabrication [66]. Here the quality factor is dominated by the side leakage and the cavity loss rate ( $\kappa_s/\kappa$ ), and  $\kappa_s/\kappa$  can be reduced by thinning down the top mirrors, which may decrease the quality factor, increase  $\kappa$ , and keep  $\kappa_s$  nearly unchanged [63]. Using this method, the quality factor  $Q \sim 17000$  ( $g/(\kappa_s + \kappa) \cong 1$ ) was achieved with  $\kappa_s/\kappa \sim 0.7$  [63].

In the condition  $g/(\kappa_s + \kappa) \cong 2.4$ ,  $\kappa_s/\kappa \sim 0$ , and  $\gamma \sim 0.1\kappa$ , the fidelity of our PCG in the odd-parity case is robust (almost 1). The fidelity in the even-parity case and efficiency of our PCG are  $F = 100\%$  and  $\eta = 98.2\%$ ,



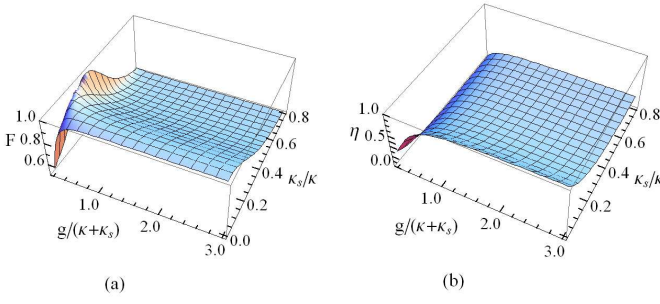


FIG. 6: The fidelity (a) and the efficiency (b) of the present parity-check gate on the polarizations of two photons for our ECP in the even parity case vs the coupling strength  $g/(\kappa + \kappa_s)$  and the side leakage rate  $\kappa_s/\kappa$  with  $\gamma = 0.1\kappa$ .

respectively. In the case  $g/(\kappa_s + \kappa) \cong 1.3$  and  $\kappa_s/\kappa \sim 0.3$ , the fidelity and efficiency of our PCG are  $F = 77.6\%$  and  $\eta = 65\%$ , respectively. For the case  $g/(\kappa_s + \kappa) \cong 1$  and  $\kappa_s/\kappa \sim 0.7$ , the fidelity and efficiency of our PCG are  $F = 66\%$  and  $\eta = 45\%$ , respectively. Therefore, the strong coupling and low cavity side leakage are required in this scheme. Our scheme is implemented with a QD-induced phase shift of  $\pm \frac{\pi}{2}$ , which requires the frequencies are adjusted to be  $\omega - \omega_c \approx \kappa/2$  ( $\omega_c = \omega_{X-}$ ). The fidelity in the even-parity case and the efficiency of our PCG vary with the coupling strength and the side leakage rate, and they are shown in Fig. 6(a) and Fig. 6(b), respectively. From these figures, one can see that our scheme is feasible in both the strong coupling regime and the weak coupling regime.  $\kappa_s$  can be made rather small by improving the sample growth or the etching process.

Compared with other ECPs for a W-class state [44–52], our ECP has some advantages. First, our ECP does not require that two of the three coefficients in the unknown W state are the same ones [45–47]. Moreover, it does not require that all of the coefficients are known for the parties, different from the ECP for W states in Ref. [48]. Second, our ECP only relies on the optical property of the quantum-dot spins inside one-sided optical microcavities, which is robust in the odd-parity instance for obtaining the standard W state. As the side leakage and cavity loss may be difficult to control or reduce for the

electron-spin qubit and photonic qubits in the double-sided QD-cavity system, our ECP is relatively easy to implement in experiment. Third, our ECP requires one of the parties to perform the local unitary operation and communicate the classical information with other parties to retain or discard their photons, which greatly simplifies the complication of classical communication. Fourth, with nonlinear optical elements, the resource can be utilized sufficiently and the total success probability of our ECP is larger than that in the conventional ECP with linear optical elements [49], which is caused by preserving the states that are discarded in the latter. With the iteration of our ECP process, the success probability  $P$  can be increased largely.

In summary, we have proposed a systematic ECP for an arbitrary unknown less-entangled three-photon W state. It has some advantages, compared with others [44–52]. First, it has a high efficiency as the parties obtain not only some partially entangled three-photon systems with two unknown parameters by picking up the robust odd-parity instance with PCG, but also some entangled two-photon systems by keeping an even-parity instance in the first step of each round of concentration, with which the parties can obtain a standard three-photon W state. Second, it is a repeatable one, which can increase the success probability largely. Third, as the side leakage and cavity loss may be difficult to control or reduce for the photonic qubits in the double-sided QD-cavity system, our ECP is relatively easier to be implemented in experiment than the ECP with a double-sided QD-cavity system [51]. These advantages maybe make our ECP more useful in quantum communication network in future.

## Acknowledgments

This work was supported by the National Natural Science Foundation of China under Grant No. 11174039 and the Open Foundation of State Key Laboratory of Networking and Switching Technology (Beijing University of Posts and Telecommunications) under Grant No. SKLNST-2013-1-13.

- 
- [1] Bennett C H, Brassard G, Crepeau C, Jozsa R, Peres A, and Wootters W K 1993 *Phys. Rev. Lett.* **70** 1895
  - [2] Bennett C H and Wiesner S J 1992 *Phys. Rev. Lett.* **69** 2881
  - [3] Liu X S, Long G L, Tong D M, and Feng L 2002 *Phys. Rev. A* **65** 022304
  - [4] Bennett C H and Brassard G 1984 In: *Proceedings of IEEE International Conference on Computers, Systems and Signal Processing*, Bangalore, India, pp. 175–179 (IEEE, New York)
  - [5] Ekert A K 1991 *Phys. Rev. Lett.* **67** 661
  - [6] Bennett C H, Brassard G, and Mermin N D 1992 *Phys. Rev. Lett.* **68** 557
  - [7] Li X H, Deng F G and Zhou H Y 2008 *Phys. Rev. A* **78** 022321
  - [8] Hillery M, Buzek V, and Berthiaume A 1999 *Phys. Rev. A* **59** 1829
  - [9] Long G L and Liu X S 2002 *Phys. Rev. A* **65** 032302
  - [10] Deng F G, Long G L, and Liu X S 2003 *Phys. Rev. A* **68** 042317
  - [11] Deng F G and Long G L 2004 *Phys. Rev. A* **69** 052319
  - [12] Bennett C H, Brassard G, Popescu S, Schumacher B, Smolin J A, and Wootters W K 1996 *Phys. Rev. Lett.* **76** 722

- [13] Pan J W, Simon C, and Zeller A 2001 *Nature* **410** 1067
- [14] Simon C and Pan J W 2002 *Phys. Rev. Lett.* **89** 257901
- [15] Sheng Y B, Deng F G, and Zhou H Y 2008 *Phys. Rev. A* **77** 042308
- [16] Sheng Y B and Deng F G 2010 *Phys. Rev. A* **81** 032307
- [17] Sheng Y B and Deng F G 2010 *Phys. Rev. A* **82** 044305
- [18] Li X H 2010 *Phys. Rev. A* **82** 044304
- [19] Deng F G 2011 *Phys. Rev. A* **83** 062316
- [20] Sheng Y B and Zhou L 2014 *Laser Phys. Lett.* **11** 085203
- [21] Sheng Y B and Zhou L 2015 *Sci. Rep.* **5** 7815
- [22] Kang, Y.H, Xia Y and Liu P M 2014 *Appl. Phys. B* **116** 977
- [23] Bennett C H, Bernstein H J, Popescu S, Schumacher B 1996 *Phys. Rev. A* **53** 2046
- [24] Yamamoto T, Koashi M, and Imoto N 2001 *Phys. Rev. A* **64** 012304
- [25] Zhao Z, Pan J W and Zhan M S 2001 *Phys. Rev. A* **64** 014301
- [26] Sheng Y B, Deng F G and Zhou H Y 2008 *Phys. Rev. A* **77** 062325
- [27] Bose S, Vedral V and Knight P L 1999 *Phys. Rev. A* **60** 194
- [28] Shi B S, Jiang Y K and Guo G C 2000 *Phys. Rev. A* **62** 054301
- [29] Sheng Y B, Zhou L, Zhao S M and Zheng B Y 2012 *Phys. Rev. A* **85** 012307
- [30] Deng F G 2012 *Phys. Rev. A* **85** 022311
- [31] Ren B C, Du F F and Deng F G 2013 *Phys. Rev. A* **88** 012302
- [32] Ren B C and Deng F G 2013 *Laser Phys. Lett.* **10** 115201
- [33] Ren B C and Long G L 2014 *Opt. Express* **22** 6547
- [34] Li X H and Ghose S 2014 *Laser Phys. Lett.* **11** 125201
- [35] Li X H, Chen X and Zeng Z 2013 *J. Opt. Soc. Am. B* **30** 2774
- [36] Li X H and Ghose S 2015 *Opt. Express* **23** 3550
- [37] Cao C, Wang C, He L Y and Zhang R 2013 *Opt. Express* **21** 4093
- [38] Wang C, Zhang Y and Jin G S 2011 *Phys. Rev. A* **84** 032307
- [39] Wang C 2012 *Phys. Rev. A* **86**, 012323 (2012)
- [40] Zhao J, Zheng C H, Shi P, Ren C N and Gu Y J 2014 *Opt. Commun.* **322** 32
- [41] Sheng Y B, Liu J, Zhao S Y, Zhou L. *Chinese Sci. Bullet.* 2013, **58** 3507
- [42] Sheng Y B and Zhou L 2013 *Entropy* **15** 1776
- [43] Xia Y, Fan L L, Hao S Y, He J, Song J, Wei R S and Huang L Q 2013 *Quantum Inf. Process.* **12** 3553
- [44] Yildiz A 2010 *Phys. Rev. A* **82** 012317
- [45] Wang H F, Zhang S and Yeon K H 2010 *J. Opt. Soc. Am. B* **27** 2159
- [46] Du F F, Li T, Ren B C, Wei H R and Deng F G 2012 *J. Opt. Soc. Am. B* **26** 1399
- [47] Gu B, Quan D H and Xiao S R 2012 *Int. J. Theor. Phys.* **51** 2966
- [48] Sheng Y B, Zhou L and Zhao S M 2012 *Phys. Rev. A* **85** 042302
- [49] Wang T J and Long G L 2013 *J. Opt. Soc. Am. B* **30** 1069
- [50] Gu B 2012 *J. Opt. Soc. Am. B* **29** 1685
- [51] Sheng Y B and Zhou L 2013 *J. Opt. Soc. Am. B* **30** 678
- [52] Fan L L, Xia Y and Song J 2014 *Quantum Inf. Process.* **13** 1967
- [53] Dür W, Vidal G and Cirac J I 2000 *Phys. Rev. A* **62** 062314
- [54] Koike S, Takahashi H, Yonezawa H, Takei N, Braunstein S L, Aoki T and Furusawa A 2006 *Phys. Rev. Lett.* **96** 060504
- [55] Cao Z L, Yang M 2004 *Physica A* **337** 132-140
- [56] Augusiak R and Horodecki P 2009 *Europhys. Lett.* **85** 50001
- [57] Lo H K and Fortescue B 2007 *Phys. Rev. Lett.* **98** 260501
- [58] Lo H K and Fortescue B 2008 *Phys. Rev. A* **78** 012348
- [59] Hu C Y, Ó'Brien J L and Munro W J 2008 *Phys. Rev. B* **78** 085307
- [60] Warburton R J, Dürr C S, Karrai K, Kotthaus J P, Medeiros-Ribeiro G and Petroff P M 1997 *Phys. Rev. Lett.* **79** 5282
- [61] Hu C Y, Ossau W, Yakovlev D R, Landwehr G, Wojtowicz T, Karczewski G and Kossut J 1998 *Phys. Rev. B* **58** R1766
- [62] Walls D F and Milburn G J 1994 *Quantum Optics* (Springer-Verlag, Berlin)
- [63] Hu C Y and Rarity J G 2011 *Phys. Rev. B* **83** 115303
- [64] Reithmaier J P, Sek G, Löffler A, Hofmann C, Kuhn S, Reitzenstein S, Keldysh L V, Kulakovskii V D, Reinecke T L and Forchel A *Nature* **432** 197
- [65] Yoshie T, Scherer A, Hendrickson J, Khitrova G, Gibbs H M, Rupper G, Ell C, Shchekin O B and Deppe D G 2004 *Nature* **432** 200
- [66] Reitzenstein, S, Hofmann, C, Gorbunov, A, Strauß, M, Kwon, S H, Schneider, C, öffler, A.L, Höfling, S, Kamp, M, and Forchel, A 2007 *Appl. Phys. Lett.* **90** 251109